

On the conformal field theories for bosonic strings in PP-waves

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ABSTRACT: Recently Kazama and Yokoi (arXiv:0801.1561 [hep-th]) have used a phase-space method to study the Virasoro algebra of type IIB superstring theory in the maximally supersymmetric R-R plane wave background in a semi-light-cone gauge. Two types of normal ordering have been considered, namely “phase space normal ordering” (PNO) and “massless normal ordering” (MNO). The second one, which is the right one to choose in flat background, has been discarded with the argument that the Virasoro algebra closes only in the first case. To understand this issue better with a completely covariant treatment we consider the easiest case of bosonic strings propagating in an arbitrary pp-wave of the simplest kind. Using the phase-space method we show that MNO is the right one to choose, at least in this case, because of the following reason. For both types of normal ordering the energy-momentum tensor satisfies the desired Virasoro algebra up to anomalous terms proportional to the space-time equation of motion of the background. However, it is MNO which gives rise to the correct spectrum - we compute the quadratic space-time action by restricting the string field inside a transverse Hilbert space. This turns out to be non-diagonal. Diagonalizing this action reproduces the spectrum directly obtained in light-cone quantization. The same method with PNO gives rise to a spectrum with negative dimensions.

KEYWORDS: Conformal Field Models in String Theory, Penrose limit and pp-wave background, Bosonic Strings.

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1. Introduction and summary

Pp-waves are certain special gravitational backgrounds for which string theory can be more tractable [1, 2]. It is interesting to understand the conformal field theories (CFT) relevant to such backgrounds in general. Because the world-sheet theory is interacting the standard canonical quantization is not straightforward. It has recently been pursued in [3] for certain NS-NS pp-waves. More recently Kazama and Yokoi (KY) [4] have applied a phase-space method to study type IIB superstring theory in the maximally supersymmetric R-R plane wave background [5]. The basic method involves imposing the equal time canonical commutation relations (after choosing the covariant gauge) to all the phase-space variables. This algebraic structure is independent of the dynamics of the theory and can be used to define a Hilbert space isomorphic to that corresponding to the flat background.¹ The dynamical questions can then be answered by writing the composite operators in terms of the phase-space variables and using the Heisenberg equation of motion. There are certain advantages of this method, at least for the present context, e.g. using this algebraic structure one can verify Virasoro algebra and establish conformal invariance [4].

There is certainly an issue of normal ordering which goes to the definition of the theory. Two types of normal ordering were considered in [4] which are called *phase-space normal ordering* (PNO) and *massless normal ordering* (MNO). It turns out that MNO defines the

¹In the usual canonical quantization method, where the Euler-Lagrange equations need to be solved, such an algebraic structure emerges after solving the theory.

theory correctly in flat background. However, it was found in [4] that PNO is the right choice for the R-R plane wave as with MNO Virasoro algebra picks up anomalous terms. It was concluded in [4] that the theory does not have a smooth flat space limit. It is, however, not very clear why such a limit should not be smooth. Moreover, according to [6] there exists a universal sector of operators whose vacuum expectation values should turn out to be same as that in flat background. This will certainly not be true if the theories in these two backgrounds are defined with different types of normal ordering.

The analysis in [4] has been done in a semi-light-cone gauge where bosons are treated covariantly, but the fermions are not. To avoid this complication and to understand the phase-space method better we consider bosonic string theory in the simplest class of pp-waves where no fluxes are switched on.² We argue that with both MNO and PNO the Virasoro algebra is satisfied up to anomalous terms that are proportional to the equation of motion for the background.³ With this result and the above argument we conclude that MNO defines the world-sheet theory correctly also for the pp-waves, at least in the present context of bosonic strings. It would be interesting to generalize this result to bosonic strings in arbitrary pp-waves which allow more general metric and include fluxes. In the context of type IIB R-R plane wave it will be interesting to see if a supersymmetric generalization of this result can be found maintaining the full covariance.

To establish further that MNO is the right choice we proceed to compute the physical spectrum of the theory by specializing to cases where the corresponding light-cone gauge fixed world-sheet theory is free. To obtain this spectrum using our CFT formulation we consider computing the closed string field theory (CSFT) [7–9] kinetic term S_2 by restricting the string field to a transverse Hilbert space relevant to the light-cone gauge. Unlike in flat case the result turns out to be non-diagonal and the spectrum is computed by diagonalizing the action. This matches precisely with the spectrum obtained directly in light-cone gauge up to terms proportional to the equation of motion for the background. The same computation gives rise to a spectrum with negative dimensions if we define the theory with PNO.

The paper is organized as follows: We analyze the Virasoro algebra for the class of pp-waves of our interest and compute the physical spectrum for the suitable cases in section 2. All these results are obtained by using MNO. We explain the problems with PNO in section 3. Some technical details are given in the appendices.

2. Simplest pp-waves with MNO

We consider closed bosonic string theory in a class of arbitrary background metric. Unless explicitly specified, we will follow the notations and conventions of [10]. When the background is an exact string solution we expect the matter part of the world-sheet theory to be a CFT such that the left and right moving Virasoro generators L_n and \tilde{L}_n respectively,

²More precisely, these pp-waves are given in eq. (2.2). Unless stated otherwise, we will always refer to these particular backgrounds as “pp-waves” in the rest of this paper.

³We present the details of the computation only for the case of MNO.

which mutually commute, satisfy the following Virasoro algebra with central charge $c = 26$.

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0} . \quad (2.1)$$

The class of backgrounds we wish to consider are the simplest pp-waves in $(d + 2)$ -dimensions:

$$ds^2 = 2dx^+ dx^- + K(x^+, \vec{x})(dx^+)^2 + d\vec{x}.d\vec{x}, \quad (2.2)$$

where $\vec{x} = (x^1, x^2, \dots, x^d)$ and $x^\pm = \frac{1}{\sqrt{2}}(x^{d+1} \pm x^0)$. It is known [1] that (2.2) is an exact string background provided:

$$d = 24, \text{ and } \vec{\partial}^2 K(x^+, \vec{x}) = 0 . \quad (2.3)$$

Therefore the matter part of the world-sheet theory should realize the Virasoro algebra as described above whenever the condition (2.3) is satisfied. This is what we will demonstrate using the phase-space formulation of [4]. We have given the details of this method for the flat background in appendix A. We follow the same steps in the present case. The components of the conjugate momentum P_μ are now given by:

$$P_- = T\partial_\tau X^+, \quad P_+ = T\left(\partial_\tau X^- + K(X^+, \vec{X})\partial_\tau X^+\right), \quad P_I = T\partial_\tau X_I . \quad (2.4)$$

Defining Π^\pm , Π^I and the corresponding right moving counterparts in the same way as in eqs. (A.3) one finds that the left and right moving components of the energy-momentum (EM) tensor take the following form in the present case:

$$\mathcal{T} = \mathcal{T}^{(0)} + \Delta\mathcal{T}, \quad \tilde{\mathcal{T}} = \tilde{\mathcal{T}}^{(0)} + \Delta\mathcal{T}, \quad (2.5)$$

where $\mathcal{T}^{(0)}$ and $\tilde{\mathcal{T}}^{(0)}$ are the EM tensor components for the flat background as given in (A.1) (we continue to use the notation (A.2)) and

$$\Delta\mathcal{T} = -\frac{1}{2}K(X^+, \vec{X})\Pi^+\tilde{\Pi}^+, \quad (2.6)$$

To quantize the system one imposes the same equal time commutation relations as in (A.4). All the equations (A.4) through (A.9) are also valid in the present context. The complete set of states in (A.9) at a fixed instant of time describes the full Hilbert space of the matter CFT.

2.1 Virasoro algebra

The Virasoro modes L_n and \tilde{L}_n for the pp-waves are defined by Fourier expanding \mathcal{T} and $\tilde{\mathcal{T}}$ in (2.5) in the similar fashion as in (A.10).⁴ Below we will use MNO, as defined in appendix A, to show that L_n and \tilde{L}_n satisfy the Virasoro algebra (2.1) with central charge $c = d + 2$ whenever the metric (2.2) satisfies the space-time equation of motion as given in (2.3).

⁴As explained at the beginning of subsection (A.1), we will be suppressing the τ dependence from all the variables.

Let us start with the left moving sector. We first notice that given the Fourier expansion for $\mathcal{T}^{(0)}(\sigma)$ as in eqs. (A.10) and the Virasoro algebra (2.1) satisfied by $L_n^{(0)}$ with $c = d + 2$, one can show that the following identity holds:

$$[\mathcal{T}^{(0)}(\sigma), \mathcal{T}^{(0)}(\sigma')] = \pi i \left[-\frac{d+2}{6} \delta'''(\sigma - \sigma') + \left(4\mathcal{T}^{(0)}(\sigma) - \frac{d+2}{6} \right) \delta'(\sigma - \sigma') + 2\partial_\sigma \mathcal{T}^{(0)}(\sigma) \delta(\sigma - \sigma') \right]. \quad (2.7)$$

We will use this observation in the reverse order to establish that L_n satisfies the same Virasoro algebra (2.1) under the required condition. This requires us to argue that $\mathcal{T}(\sigma) = \mathcal{T}^{(0)}(\sigma) + \Delta\mathcal{T}(\sigma)$ also satisfies the same commutation relation in (2.7) with $\mathcal{T}^{(0)}(\sigma)$ replaced by $\mathcal{T}(\sigma)$. It has been shown in appendix B that,

$$[\Delta\mathcal{T}(\sigma), \Delta\mathcal{T}(\sigma')] = 0. \quad (2.8)$$

We therefore need to establish,

$$[\mathcal{T}^{(0)}(\sigma), \Delta\mathcal{T}(\sigma')] + [\Delta\mathcal{T}(\sigma), \mathcal{T}^{(0)}(\sigma')] = 4\pi i \Delta\mathcal{T}(\sigma) \delta'(\sigma - \sigma') + 2\pi i \partial_\sigma \Delta\mathcal{T}(\sigma) \delta(\sigma - \sigma'), \quad (2.9)$$

whenever,

$$\begin{aligned} \bar{\partial}^2 K(\sigma) &\equiv \bar{\partial}^2 K(X^+(\sigma), \vec{X}(\sigma)) \\ &= \int \frac{dk^- d\vec{k}}{(2\pi)^{(d+1)/2}} \tilde{K}(k^-, \vec{k}) \vec{k}^2 : e^{i(k^- X^+ + \vec{k} \cdot \vec{X})}(\sigma) : \\ &= 0, \end{aligned} \quad (2.10)$$

where $\tilde{K}(k^-, \vec{k})$ is the Fourier transform of $K(x^+, \vec{x})$ and $: :$ is the massless normal ordering. This is indeed true as shown in appendix B.

The analysis for the right moving sector proceeds in the similar way. In this case $\tilde{\mathcal{T}}^{(0)}(\sigma)$ satisfies:

$$[\tilde{\mathcal{T}}^{(0)}(\sigma), \tilde{\mathcal{T}}^{(0)}(\sigma')] = \pi i \left[\frac{d+2}{6} \delta'''(\sigma - \sigma') - \left(4\mathcal{T}^{(0)}(\sigma) - \frac{d+2}{6} \right) \delta'(\sigma - \sigma') - 2\partial_\sigma \mathcal{T}^{(0)}(\sigma) \delta(\sigma - \sigma') \right]. \quad (2.11)$$

Therefore we need to show:

$$[\tilde{\mathcal{T}}^{(0)}(\sigma), \Delta\mathcal{T}(\sigma')] + [\Delta\mathcal{T}(\sigma), \tilde{\mathcal{T}}^{(0)}(\sigma')] = -4\pi i \Delta\mathcal{T}(\sigma) \delta'(\sigma - \sigma') - 2\pi i \partial_\sigma \Delta\mathcal{T}(\sigma) \delta(\sigma - \sigma'), \quad (2.12)$$

whenever (2.10) is satisfied. This can be proved following the same procedure as in B.2.

Finally, we need to show that the left and right moving Virasoro generators commute with each other. We have shown in appendix B that indeed,

$$[\mathcal{T}(\sigma), \tilde{\mathcal{T}}(\sigma')] = 0, \quad (2.13)$$

whenever the condition (2.10) is satisfied.

2.2 Physical spectrum

In this section we will use our CFT method to reproduce the physical spectrum of strings obtained in light-cone gauge by specializing to the following cases:

$$K(x^+, \vec{x}) = - \sum_I s_I x^I x^I, \text{ such that } \sum_I s_I = 0. \quad (2.14)$$

The second equation is the space-time equation of motion in (2.3) for the present ansatz. The light-cone quantization can be done following [10]. In this case the transverse coordinate X^I becomes a massive scalar field with mass squared [5]:

$$m_I^2 = s_I (\alpha' p^+)^2, \quad (2.15)$$

p^+ being the light-cone momentum.⁵ The light-cone hamiltonian H_{lc} turns out to satisfy:

$$2p^+ H_{lc} + \vec{p}^2 + \sum_I s_I X_0^I X_0^I (p^+)^2 + \frac{2}{\alpha'} \left[\sum_{n>0, I} (\alpha_{-n}^I \alpha_n^I + \tilde{\alpha}_{-n}^I \tilde{\alpha}_n^I) + \mathcal{C}(m_I^2) \right] = 0, \quad (2.16)$$

where the non-trivial commutators of the oscillators are given by,

$$[\alpha_n^I, \alpha_m^J] = [\tilde{\alpha}_n^I, \tilde{\alpha}_m^J] = w_n^I \delta^{IJ} \delta_{n, -m}, \quad w_n^I = n \sqrt{1 + \frac{m_I^2}{n^2}}, \quad (2.17)$$

and the “zero point energy” $\mathcal{C}(m_I^2)$ is given by the total Casimir energy of all the massive scalar fields:

$$\mathcal{C}(m_I^2) = \sum_{n>0, I} w_n^I. \quad (2.18)$$

The expression (2.16) reduces to the correct one for flat background in the limit $s_I \rightarrow 0$. In this case the zero point energy reduces to [10]:

$$\lim_{s_I \rightarrow 0} \mathcal{C}(m_I^2) \rightarrow d \sum_{n>0} n = -2, \quad (2.19)$$

where use has been made of the Riemann zeta function: $\zeta(-1) = \sum_{n>0} n = -\frac{1}{12}$.

We will now derive an expression for the light-cone spectrum using our CFT approach and see that the result is indeed the same up to a term proportional to the space-time equation of motion in (2.14). To do that we consider computing the CSFT kinetic term [7–9]. Given any matter CFT of central charge $c = 26$ this is given by [8],

$$S_2 = -\frac{1}{2} \langle \Psi_{b_{pz}} | c_0^- Q_B | \Psi \rangle, \quad (2.20)$$

⁵Certainly, due to the condition in (2.14), it is necessary that some of the directions have negative mass squared. This implies that the system is unstable. This is a property of the background, chosen for simplicity, and can be bypassed by switching on fluxes. However, all the non-zero modes X_n^I 's (A.5), which we will be mainly concerned with, can be stabilized by keeping $|m_I^2| \leq 1$. In any case, here we are focussing on a method of analyzing string theory in pp-waves for which we may ignore such issues.

where $c_0^- = (c_0 - \tilde{c}_0)/2$ and Q_B is the total BRST operator.⁶ $|\Psi\rangle$ is the string field given by an arbitrary ghost number 2 state in the combined matter-ghost CFT satisfying:

$$(b_0 - \tilde{b}_0)|\Psi\rangle = (L_0^T - \tilde{L}_0^T)|\Psi\rangle = 0, \tag{2.21}$$

where L_0^T and \tilde{L}_0^T are the total (matter + ghost) Virasoro zero modes. $\langle\Psi_{bpz}|$ is the BPZ conjugate of $|\Psi\rangle$ as defined in [9].

Given the above definition of S_2 we will now discuss how to compute it using the phase-space method in the present context. Let us first consider the string field $|\Psi\rangle$. Using eqs. (2.5) one can see that the subsidiary conditions in (2.21) are same as that in flat background. Given this and the isomorphism between the CFT Hilbert spaces corresponding to flat and pp-wave backgrounds we conclude that $|\Psi\rangle$ can be expanded in terms of the CFT states precisely in the same way as in flat case. Next we consider the computation of $\langle\Psi_{bpz}|$. Usually BPZ conjugate of a state is defined through a conformal transformation given by the inversion map⁷ I applied on the relevant vertex operator [9]. In our case it is convenient to view I as the combined PT transformation on the world-sheet with Minkowski signature:

$$I \circ \tau = -\tau, \quad I \circ \sigma = -\sigma. \tag{2.22}$$

Its action on the phase-space modes can be easily determined to be:⁸

$$I \circ \Pi_n^\mu(\tau) = -\Pi_{-n}^\mu(\tau), \quad I \circ \tilde{\Pi}_n^\mu(\tau) = -\tilde{\Pi}_{-n}^\mu(\tau), \tag{2.23}$$

where we have explicitly displayed the τ dependence. Given any state $|\phi, k\rangle = \phi|k\rangle$, where ϕ contains all the matter and ghost creation operators, the BPZ conjugate is given by: $\langle k, \phi_{bpz}| = \langle -k|I \circ \phi$, where $\langle k|$ is the hermitian conjugate of $|k\rangle$, implying: $\langle k|p\rangle \propto \delta^{d+2}(k-p)$. Noticing that eqs. (2.23) are valid both in flat background and pp-waves we arrive at the following conclusion: In flat background the following relation holds [9]:

$$\langle\Psi_{bpz}| = -\langle\Psi|, \tag{2.24}$$

$\langle\Psi|$ being the hermitian conjugate of $|\Psi\rangle$, whenever the string field components satisfy the correct reality property. This relation is still valid in pp-waves if the string field components satisfy the same reality condition. This enables us to write,

$$S_2 = \frac{1}{2} \langle\Psi|c_0^- Q|\Psi\rangle, \tag{2.25}$$

also for the pp-waves.⁹

⁶ Q_B is given by the same oscillator expression as in [10] with the matter Virasoro generators replaced by the ones defined in subsection 2.1 for pp-waves.

⁷On the euclidean world-sheet parameterized by (τ_E, σ) , τ_E being the euclidean time, the inversion map is given by $I \circ z = \frac{1}{z}$, where $z = \exp(\tau_E - i\sigma)$.

⁸The action of I on the ghost sector is taken to be the same as given in [9].

⁹Notice that in the phase-space formulation the matter part (A.9) of a CFT state is τ dependent. But S_2 itself is τ independent as expected. The reason is simply the fact that the matter part of any term in S_2 can be written as the vacuum expectation value of a collection of phase-space modes at the same time. Such an expectation value can be computed entirely by using the equal-time commutators in (A.6).

For the purpose of computing the light-cone spectrum we will now compute S_2 following eq. (2.25) by restricting the string field to the relevant transverse Hilbert space \mathcal{H}_\perp .¹⁰ \mathcal{H}_\perp is spanned by the following basis states which satisfy the $(b_0 - \tilde{b}_0)$ -condition in (2.21),

$$\begin{aligned} |\{N_{In}\}, \{\tilde{N}_{In}\}, p\rangle &\equiv c_1 \tilde{c}_1 |\{N_{In}\}, \{\tilde{N}_{In}\}, p\rangle, \\ |\{N_{In}\}, \{\tilde{N}_{In}\}, p\rangle &\equiv \prod_{n>0, I} \frac{(\Pi_{-n}^I)^{N_{In}} (\tilde{\Pi}_{-n}^I)^{\tilde{N}_{In}}}{\sqrt{n^{N_{In} + \tilde{N}_{In}} N_{In}! \tilde{N}_{In}!}} |p\rangle, \end{aligned} \quad (2.26)$$

The restricted string field is therefore expanded as:

$$|\Psi_\perp\rangle = \sum'_{\{N_{In}\}, \{\tilde{N}_{In}\}} \int dk \tilde{\psi}_{\{N_{In}\}, \{\tilde{N}_{In}\}}(k) |\{N_{In}\}, \{\tilde{N}_{In}\}, p\rangle, \quad (2.27)$$

where the prime on the summation refers to the restriction $\sum_{n>0, I} n(N_{In} - \tilde{N}_{In}) = 0$ coming from the $(L_0^T - \tilde{L}_0^T)$ -condition in (2.21). Following the standard computation the quadratic action is evaluated to be,

$$S_2 \propto \sum'_{\{N'_{In}\}, \{\tilde{N}'_{In}\}} \sum'_{\{N_{In}\}, \{\tilde{N}_{In}\}} \int dp' dp \tilde{\psi}_{\{N'_{In}\}, \{\tilde{N}'_{In}\}}^*(p') \tilde{\psi}_{\{N_{In}\}, \{\tilde{N}_{In}\}}(p) \langle p', \{N'_{In}\}, \{\tilde{N}'_{In}\} | \mathcal{S}_2 | \{N_{In}\}, \{\tilde{N}_{In}\}, p \rangle, \quad (2.28)$$

where $\tilde{\psi}_{\{N_{In}\}, \{\tilde{N}_{In}\}}^*(p)$ is the complex conjugate of $\tilde{\psi}_{\{N_{In}\}, \{\tilde{N}_{In}\}}(p)$, $\langle p, \{N_{In}\}, \{\tilde{N}_{In}\} |$ is the hermitian conjugate of $|\{N_{In}\}, \{\tilde{N}_{In}\}, p\rangle$ and

$$\begin{aligned} \mathcal{S}_2 &= L_0 + \tilde{L}_0 - 2 = \mathcal{S}_2^0 + \mathcal{S}_2^{\neq 0}, \\ \mathcal{S}_2^0 &= 2p^+ p^- + p^2 + \sum_I s_I X_0^I X_0^I (p^+)^2, \\ \mathcal{S}_2^{\neq 0} &= \frac{2}{\alpha'} \sum_{n>0, I} \left\{ \left(1 + \frac{m_I^2}{2n^2} \right) (\Pi_{-n}^I \Pi_n^I + \tilde{\Pi}_{-n}^I \tilde{\Pi}_n^I) - \frac{m_I^2}{2n^2} (\Pi_n^I \tilde{\Pi}_n^I + \Pi_{-n}^I \tilde{\Pi}_{-n}^I) \right\} - \frac{4}{\alpha'}, \end{aligned} \quad (2.29)$$

where the second and third lines display the contribution of $(L_0 + \tilde{L}_0 - 2)$ that is nonzero inside \mathcal{H}_\perp . Obviously the quadratic action is not diagonal in the basis that we are working with. To diagonalize this we relate the Π -oscillators and the α -oscillators in (2.17) in the following way:

$$\Pi_n^I = \frac{1}{2} (M_{In}^+ \alpha_n^I + M_{In}^- \tilde{\alpha}_{-n}^I), \quad \tilde{\Pi}_n^I = \frac{1}{2} (M_{In}^+ \tilde{\alpha}_n^I + M_{In}^- \alpha_{-n}^I), \quad \forall n \neq 0, \quad (2.30)$$

where, $M_{In}^\pm = 1 \pm \frac{n}{w_n}$. One can check that $\mathcal{S}_2^{\neq 0}$ is diagonal in terms of the α -oscillators and it takes the following form

$$\mathcal{S}_2^{\neq 0} = \frac{2}{\alpha'} \left[\sum_{n>0, I} (\alpha_{-n}^I \alpha_n^I + \tilde{\alpha}_{-n}^I \tilde{\alpha}_n^I) + \hat{C}(m_I^2) \right], \quad (2.31)$$

¹⁰Since the standard light-cone gauge can be fixed on the world-sheet, it is expected that all the BRST cohomology elements can be found in this transverse space. See also [4].

The light-cone hamiltonian H_{lc} is identified with p^- in the second equation of (2.29) and therefore is given by the same expression (2.16) with $\mathcal{C}(m_I^2)$ replaced by $\hat{\mathcal{C}}(m_I^2)$ which is found to be,

$$\begin{aligned} \hat{\mathcal{C}}(m_I^2) &= -2 + \frac{1}{2} \sum_{n>0, I} w_n^I \left[\left(1 + \frac{m_I^2}{2n^2} \right) (M_{In}^-)^2 - \frac{m_I^2}{2n^2} M_{In}^+ M_{In}^- \right], \\ &= -2 - d \sum_{n>0} n + \mathcal{C}(m_I^2) - \frac{1}{2} \left(\sum_I m_I^2 \right) \sum_{n>0} \frac{1}{n}, \end{aligned} \quad (2.32)$$

The first two terms in the second line cancel according to (2.19). The last term drops off when the space-time equation of motion in (2.14) is satisfied, in which case $\hat{\mathcal{C}}(m_I^2) = \mathcal{C}(m_I^2)$.

Certainly the quadratic action takes the diagonal form in terms of a new set of fields $\tilde{\chi}_{\{N_{In}\}, \{\tilde{N}_{In}\}}(p)$ which correspond to the basis states $|\{N_{In}\}, \{\tilde{N}_{In}\}, k\rangle \equiv c_1 \tilde{c}_1 |\{N_{In}\}, \{\tilde{N}_{In}\}, k\rangle$, where $|\{N_{In}\}, \{\tilde{N}_{In}\}, k\rangle$ is given by the second equation in (2.26) with all the Π oscillators replaced by the corresponding α oscillators and $|k\rangle$ replaced by the new vacuum $|k\rangle$ defined by,

$$\alpha_n^I |k\rangle = \tilde{\alpha}_n^I |k\rangle = 0, \quad \forall n > 0, \quad (2.33)$$

Given this definition and eqs. (2.30) it is clear that $|k\rangle$ is given by a squeezed state in the Fock space over $|k\rangle$ (and *vice versa*) with a normalization constant that can be fixed by demanding: $\langle k'|k\rangle = \langle\langle k'|k\rangle\rangle$. Both the basis $|\{N_{In}\}, \{\tilde{N}_{In}\}, k\rangle$ and $|\{N_{In}\}, \{\tilde{N}_{In}\}, k\rangle$ are orthonormal and complete. Using this one can explicitly relate the two sets of fields $\tilde{\psi}_{\{N_{In}\}, \{\tilde{N}_{In}\}}(k)$ and $\tilde{\chi}_{\{N_{In}\}, \{\tilde{N}_{In}\}}(k)$. However we will not need those expressions for the purpose of the present work.

3. Problem with PNO

The phase-space normal ordering [4] is given by identifying X_n^μ 's with $n > 0$ and $P_{\mu n}$'s with $n \geq 0$ as annihilation operators. Using the relation (A.3) and the Fourier expansions (A.5) we conclude that the corresponding ground state $|0\rangle'$ satisfies:

$$\Pi_n^\mu |0\rangle' = \tilde{\Pi}_{-n}^\mu |0\rangle' = 0, \quad \forall n \geq 0. \quad (3.1)$$

Therefore $|0\rangle'$ differs from $|0\rangle$ in the right moving sector. By redefining the composite operators with PNO and repeating the similar analysis of subsection 2.1 and appendix B one can show that the EM tensor components satisfy the required Virasoro algebra up to anomalous terms proportional to the equation of motion for the background. We will not discuss this in any further detail. Instead we will show in this section that such a normal ordering gives rise to an incorrect spectrum.

To see this let us first consider the flat background. Since $L_n^{(0)}$ and $\tilde{L}_n^{(0)}$ do not involve any normal ordering for $n \neq 0$, they are still given by the same expressions as in the case of MNO. Therefore we have,

$$L_n^{(0)} |0\rangle' = \tilde{L}_{-n}^{(0)} |0\rangle' = 0, \quad \forall n \geq 0. \quad (3.2)$$

Certainly this is not the correct choice of the vacuum. In particular, the excited states (right moving sector) are obtained by applying $\tilde{L}_n^{(0)}$'s ($n > 0$) on $|0\rangle'$ and have negative conformal dimensions. Notice that $|0\rangle'$ can not be constructed in the Fock space built over $|0\rangle$ and *vice versa* as the latter contains states of positive dimensions only.

The fact that PNO is a wrong choice for the flat background already tells us that it does not work for the pp-wave background, as it does not lead to the right flat space limit. Another way to see this is the following: There exists a universal sector of operators [6] to which the background appears to be flat. This set of operators corresponds to a subspace of the full Hilbert space whose overlap with \mathcal{H}_\perp is given by all the states in \mathcal{H}_\perp with zero light-cone momentum $p^+ = 0$. It is a necessary condition that the matrix elements of the “universal operators” between the states inside this overlap should turn out to be same as that computed in flat background. This condition is certainly not satisfied if we define the theory with different normal ordering for flat background and pp-waves.

We will now show that if we compute S_2 following the same steps of subsection 2.2 then, just like in flat space, we get a spectrum of negative dimensions. Now the transverse sector \mathcal{H}_\perp has to be spanned by the basis states $|\{N_{In}\}, \{\tilde{N}_{In}\}, k\rangle' = c_1 \tilde{c}_1 |\{N_{In}\}, \{\tilde{N}_{In}\}, k\rangle'$, where the states $|\{N_{In}\}, \{\tilde{N}_{In}\}, k\rangle'$ are obtained by replacing $\tilde{\Pi}_{-n}^I \rightarrow \tilde{\Pi}_n^I$ and $|k\rangle \rightarrow |k\rangle' = e^{ik \cdot X_0} |0\rangle'$ in the last equation of (2.26).¹¹ The result for S_2 is given by eq. (2.28) with the old basis replaced by the new ones. For S_2 , the first two equations in (2.29) still hold. But the non-zero-mode-contribution $S_2^{\neq 0}$ is given by the same expression in (2.29) with opposite ordering for the $\tilde{\Pi}$ oscillators, which is expected, given eqs. (3.1). Diagonalizing such an expression one gets:

$$S_2^{\neq 0} = \frac{2}{\alpha'} \sum_{n>0, I} (\alpha_{-n}^I \alpha_n^I + \tilde{\alpha}_n^I \tilde{\alpha}_{-n}^I) - \frac{4}{\alpha'}, \tag{3.3}$$

Notice again that the $\tilde{\alpha}$ -oscillators are oppositely ordered with respect to the α -oscillators. This is because given eqs. (3.1) and (2.30) one concludes,

$$\alpha_n^I |k\rangle' = \tilde{\alpha}_{-n}^I |k\rangle' = 0, \quad \forall n > 0. \tag{3.4}$$

Given the commutators in (2.17), it is clear that $S_2^{\neq 0}$ in (3.3) has negative eigenvalues.¹² We emphasize that the operators in (3.3) can not be normal ordered with respect to the α -vacuum $|k\rangle\rangle$ defined in (2.33), as was done in [4]. This is simply because the states $|k\rangle'$ and $|k\rangle\rangle$ can not be constructed in the Fock space over each other.

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A. Phase-space formulation for flat background

To introduce the phase-space formulation of [4] and to set up our notations we work out the details for the flat background here. We will simply draw a parallel when we study

¹¹The state $|\{N_{In}\}, \{\tilde{N}_{In}\}, k\rangle'$, therefore, has a negative norm whenever $\sum_{n>0, I} \tilde{N}_{In}$ is odd.

¹²In fact both L_0 and \tilde{L}_0 separately have negative eigenvalues.

pp-waves in section 2. For certain quantities we use the superscript (0) to refer to the flat background. The same notations without this superscript will refer to the pp-waves. All the equations in this appendix where the two sides are related by \doteq will also be valid for pp-waves with the superscript (0) removed wherever needed.

The world-sheet action for the flat background is: $S^{(0)} = -\frac{T}{2} \int d\tau d\sigma \sqrt{-g} g^{ab} \eta_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$, where $T = \frac{1}{2\pi\alpha'}$ is the string tension and $\mu, \nu = 0, 1, \dots, d+1$. The energy-momentum (EM) tensor is defined to be: $\mathcal{T}_{ab}^{(0)}(\tau, \sigma) \doteq -\frac{4\pi}{\sqrt{-g}} \frac{\delta S^{(0)}}{\delta g^{ab}(\tau, \sigma)}$. After fixing the covariant gauge: $g_{ab} = \eta_{ab}$ the left and the right moving components of the EM tensor take the following form:

$$\begin{aligned} \mathcal{T}^{(0)} &\doteq \frac{1}{2}(\mathcal{T}_{\tau\tau} + \mathcal{T}_{\tau\sigma}) = \frac{1}{2}\Pi \cdot \Pi, \\ \tilde{\mathcal{T}}^{(0)} &\doteq \frac{1}{2}(\tilde{\mathcal{T}}_{\tau\tau} - \tilde{\mathcal{T}}_{\tau\sigma}) = \frac{1}{2}\tilde{\Pi} \cdot \tilde{\Pi}, \end{aligned} \tag{A.1}$$

where we have used,

$$A \cdot B \doteq A^+ B^- + A^- B^+ + \vec{A} \cdot \vec{B}, \tag{A.2}$$

with the \pm components defined below eq. (2.2) and

$$\Pi^\mu \doteq \sqrt{\frac{\pi}{T}} \eta^{\mu\nu} P_\nu + \sqrt{\pi T} \partial_\sigma X^\mu, \quad \tilde{\Pi}^\mu \doteq \sqrt{\frac{\pi}{T}} \eta^{\mu\nu} P_\nu - \sqrt{\pi T} \partial_\sigma X^\mu, \tag{A.3}$$

where $P_\mu \doteq \frac{\delta S^{(0)}}{\delta \partial_\tau X^\mu}$ is the momentum conjugate to X^μ . Notice that Π^μ and $\tilde{\Pi}^\mu$ are defined using $\eta^{\mu\nu}$ also for the pp-waves.

A.1 Quantization with MNO

In the phase-space method all the world-sheet fields and their conjugate momenta are Fourier expanded along the σ direction at a given instant of time τ . The complete operator algebra is also constructed at a given instant of time. This is sufficient to construct the Hilbert space. To obtain the time evolution of any operator one uses the Heisenberg equation of motion. Now onwards we will suppress the τ dependence of all the fields and it is understood that all the following analysis are done at a fixed instant of time. The equal time canonical commutation relations are:

$$[X^\mu(\sigma), P_\nu(\sigma')] \doteq i\delta_\nu^\mu \delta(\sigma - \sigma'). \tag{A.4}$$

The Fourier expansion of various fields are given by,

$$\begin{aligned} X^\mu(\sigma) &\doteq \sum_{n \in \mathbb{Z}} X_n^\mu e^{-in\sigma}, & P^\mu(\sigma) &\doteq \sum_{n \in \mathbb{Z}} P_n^\mu e^{-in\sigma}, \\ \Pi^\mu(\sigma) &\doteq \sum_{n \in \mathbb{Z}} \Pi_n^\mu e^{-in\sigma}, & \tilde{\Pi}^\mu(\sigma) &\doteq \sum_{n \in \mathbb{Z}} \tilde{\Pi}_n^\mu e^{in\sigma}, \end{aligned} \tag{A.5}$$

implying the hermiticity property: $(A_n^\mu)^\dagger \doteq A_{-n}^\mu$, where A_n^μ stands for any of the Fourier modes appearing in the above equations and the following non-trivial commutators:

$$[X_m^\mu, P_{\nu n}] \doteq \frac{i}{2\pi} \delta_\nu^\mu \delta_{m+n,0}, \quad [\Pi_m^\mu, \Pi_n^\nu] \doteq [\tilde{\Pi}_m^\mu, \tilde{\Pi}_n^\nu] \doteq m\eta^{\mu\nu} \delta_{m+n,0}. \tag{A.6}$$

Therefore the complete set of operators are:¹³

$$\begin{aligned} \text{zero modes: } & X_0^\mu, \quad \eta^{\mu\nu} P_{\nu 0} \doteq \sqrt{\frac{T}{\pi}} \Pi_0^\mu \doteq \sqrt{\frac{\tilde{T}}{\pi}} \tilde{\Pi}_0^\mu, \\ \text{non-zero modes: } & \Pi_n^\mu, \quad \tilde{\Pi}_n^\mu, \quad n \neq 0. \end{aligned} \quad (\text{A.7})$$

The massless normal ordering is defined by identifying Π_n^μ and $\tilde{\Pi}_n^\mu$ for $n \geq 0$ as annihilation operators. As it is well known that the corresponding ground state is indeed the $\text{SL}(2, \mathbb{C})$ invariant vacuum $|0\rangle$, i.e.,

$$\Pi_n^\mu |0\rangle \doteq \tilde{\Pi}_n^\mu |0\rangle \doteq 0, \quad \forall n \geq 0. \quad (\text{A.8})$$

The excited states are given by:

$$(\Pi_{-n_1}^{\mu_1} \Pi_{-n_2}^{\mu_2} \cdots)(\tilde{\Pi}_{-m_1}^{\nu_1} \tilde{\Pi}_{-m_2}^{\nu_2} \cdots)|k\rangle, \quad (\text{A.9})$$

where $|k\rangle \doteq e^{ik \cdot X_0} |0\rangle$. The Virasoro generators $L_n^{(0)}$ and $\tilde{L}_n^{(0)}$, which satisfy the Virasoro algebra (2.1) with central charge $c = d + 2$, are defined by,

$$\mathcal{T}^{(0)}(\sigma) \doteq \sum_{n \in \mathbb{Z}} L_n^{(0)} e^{-in\sigma}, \quad \tilde{\mathcal{T}}^{(0)}(\sigma) \doteq \sum_{n \in \mathbb{Z}} \tilde{L}_n^{(0)} e^{in\sigma}. \quad (\text{A.10})$$

The time evolution of various operators can be obtained by using the Heisenberg equation of motion:

$$\frac{d}{d\tau} \mathcal{O} \doteq i[\mathcal{O}, H^{(0)}] + \frac{\partial}{\partial \tau} \mathcal{O}, \quad (\text{A.11})$$

where $H^{(0)} \doteq 2\pi(L_0^{(0)} + \tilde{L}_0^{(0)})$ is the hamiltonian. Using this one can show:

$$\Pi^\mu(\tau, \sigma) = \sum_{n \in \mathbb{Z}} \Pi_n^\mu(0) e^{in(2\pi\tau - \sigma)}, \quad \tilde{\Pi}^\mu(\tau, \sigma) = \sum_{n \in \mathbb{Z}} \tilde{\Pi}_n^\mu(0) e^{in(2\pi\tau + \sigma)}, \quad (\text{A.12})$$

$$\mathcal{T}^{(0)}(\tau, \sigma) \doteq \sum_{n \in \mathbb{Z}} L_n^{(0)}(0) e^{in(2\pi\tau - \sigma)}, \quad \tilde{\mathcal{T}}^{(0)}(\tau, \sigma) \doteq \sum_{n \in \mathbb{Z}} \tilde{L}_n^{(0)}(0) e^{in(2\pi\tau + \sigma)}, \quad (\text{A.13})$$

where we have shown the time dependence explicitly. Notice that the ‘‘holomorphicity’’ properties of the fields Π^μ and $\tilde{\Pi}^\mu$ in eqs. (A.12) will not hold for pp-waves as the corresponding modes will have more complicated commutation relations with the hamiltonian. However, the same for $\mathcal{T}^{(0)}$ and $\tilde{\mathcal{T}}^{(0)}$ are obtained using the Virasoro algebra (2.1). Therefore eqs. (A.13) (with the superscript (0) removed) will be valid also in pp-waves.

B. Relevant commutators

Below we prove equations (2.8), (2.9), (2.13) required to establish the Virasoro algebra in subsection 2.1.

¹³For the non-zero modes the Π -oscillators defined here are same as the standard α -oscillators (with the τ -evolution included) conventionally used [10] for string quantization in flat background. The zero modes are related by a proportionality constant.

B.1 Proof of eq. (2.8)

The normal ordered expression for $\Delta\mathcal{T}(\sigma)$ is given by:

$$\Delta\mathcal{T}(\sigma) = -\frac{1}{2}K\Pi^+\tilde{\Pi}^+(\sigma), \quad (\text{B.1})$$

where,

$$\begin{aligned} K(\sigma) &\equiv : K(X^+(\sigma), \vec{X}(\sigma)) :, \\ &= \int \frac{dk^- d\vec{k}}{(2\pi)^{(d+1)/2}} \tilde{K}(k^-, \vec{k}) e^{ik^- X^+(\sigma)} : e^{i\vec{k}\cdot\vec{X}} :(\sigma), \end{aligned} \quad (\text{B.2})$$

and

$$: e^{i\vec{k}\cdot\vec{X}} :(\sigma) = e^{i\vec{k}\cdot\vec{X}_0} E(\vec{k}; \sigma) \tilde{E}(\vec{k}; \sigma), \quad (\text{B.3})$$

where,

$$\begin{aligned} E(\vec{k}; \sigma) &= E_-(\vec{k}; \sigma) E_+(\vec{k}; \sigma), \\ E_{\pm}(\vec{k}; \sigma) &= \exp\left(\mp \frac{1}{2\sqrt{\pi T}} \sum_{n \geq 1} \frac{1}{n} \vec{k}\cdot\vec{\Pi}_{\pm n} e^{\mp in\sigma}\right). \end{aligned} \quad (\text{B.4})$$

The operator $\tilde{E}(\vec{k}; \sigma)$ takes the similar form as above with $\Pi_{\pm n}^I$ replaced by $\tilde{\Pi}_{\pm n}^I$ and σ replaced by $-\sigma$. To evaluate the commutator $[\Delta\mathcal{T}(\sigma), \Delta\mathcal{T}(\sigma')]$ one needs to compute $[: e^{i\vec{k}\cdot\vec{X}}(\sigma) :, : e^{i\vec{p}\cdot\vec{X}}(\sigma') :]$. The result turns out to be zero as can be shown by using:

$$E(\vec{k}; \sigma) \tilde{E}(\vec{k}; \sigma) E(\vec{p}; \sigma') \tilde{E}(\vec{p}; \sigma') =: E(\vec{k}; \sigma) \tilde{E}(\vec{k}; \sigma) E(\vec{p}; \sigma') \tilde{E}(\vec{p}; \sigma') : e^{\vec{k}\cdot\vec{p}D(\sigma-\sigma')}, \quad (\text{B.5})$$

where $D(\sigma)$ is an even function of σ . This establishes eq. (2.8).

B.2 Proof of eq. (2.9)

Using the expressions in (2.5), (A.1) and (2.6) we may write:

$$\begin{aligned} [\mathcal{T}^{(0)}(\sigma), \Delta\mathcal{T}(\sigma')] &= -\frac{1}{2}[: \Pi^+ \Pi^-(\sigma) :, K\Pi^+ \tilde{\Pi}^+(\sigma')] \\ &\quad -\frac{1}{4}[: \vec{\Pi}\cdot\vec{\Pi}(\sigma) :, K\Pi^+ \Pi^+(\sigma')]. \end{aligned} \quad (\text{B.6})$$

The normal ordered quadratic operators are given by,

$$: \Pi^\mu \Pi^\nu(\sigma) := \Pi_{(+)}^\mu \Pi_{(+)}^\nu(\sigma) + \Pi_{(-)}^\mu \Pi_{(-)}^\nu(\sigma) + \Pi_{(-)}^\mu \Pi_{(+)}^\nu(\sigma) + \Pi_{(-)}^\nu \Pi_{(+)}^\mu(\sigma), \quad (\text{B.7})$$

where following [4] we have defined,

$$\Pi_{(+)}^\mu(\sigma) = \sum_{n \geq 0} \Pi_n^\mu e^{-in\sigma}, \quad \Pi_{(-)}^\mu(\sigma) = \sum_{n > 0} \Pi_{-n}^\mu e^{in\sigma}. \quad (\text{B.8})$$

Given the above definitions the commutators in eq. (2.12) can be computed using eqs. (A.6) and the techniques used in [4]. The final result is found to be,

$$\begin{aligned}
 [\mathcal{T}^{(0)}(\sigma), \Delta\mathcal{T}(\sigma')] &= \left(-\pi i K(\sigma') \Pi^+(\sigma) \tilde{\Pi}^+(\sigma') + \frac{i}{8T} \vec{\partial}^2 K \Pi^+ \tilde{\Pi}^+(\sigma') \right) \delta'(\sigma - \sigma') \\
 &+ \sqrt{\frac{\pi}{T}} \left(\frac{i}{2} \partial_+ K (\Pi^+)^2 \tilde{\Pi}^+(\sigma) + \frac{i}{2} : \vec{\Pi} \cdot \vec{\partial} K : \Pi^+ \tilde{\Pi}^+(\sigma) \right. \\
 &\quad \left. + \frac{1}{8\sqrt{\pi T}} \vec{\partial}^2 K \Pi^+ \tilde{\Pi}^+(\sigma) \right) \delta(\sigma - \sigma'), \tag{B.9}
 \end{aligned}$$

where the derivatives of K are defined following the same way as in (2.10). Using this result and the identity [4],

$$\mathcal{O}(\sigma') \delta'(\sigma - \sigma') = \mathcal{O}(\sigma) \delta'(\sigma - \sigma') + \partial_\sigma \mathcal{O}(\sigma) \delta(\sigma - \sigma'), \tag{B.10}$$

one finds:

$$\begin{aligned}
 [\mathcal{T}^{(0)}(\sigma), \Delta\mathcal{T}(\sigma')] + [\Delta\mathcal{T}(\sigma), \mathcal{T}^{(0)}(\sigma')] &= 4\pi i \Delta\mathcal{T}(\sigma) \delta'(\sigma - \sigma') + 2\pi i \partial_\sigma \Delta\mathcal{T}(\sigma) \delta(\sigma - \sigma') \\
 &\quad - \frac{i}{4T} \left(2\vec{\partial}^2 \Delta\mathcal{T}(\sigma) \delta'(\sigma - \sigma') \right. \\
 &\quad \left. + \partial_\sigma \vec{\partial}^2 \Delta\mathcal{T}(\sigma) \delta(\sigma - \sigma') \right), \tag{B.11}
 \end{aligned}$$

where

$$\vec{\partial}^2 \Delta\mathcal{T}(\sigma) = -\frac{1}{2} \vec{\partial}^2 K \Pi^+ \tilde{\Pi}^+(\sigma). \tag{B.12}$$

This establishes eqs. (2.9), (2.10).

B.3 Proof of eq. (2.13)

Using $[\mathcal{T}^{(0)}(\sigma), \tilde{\mathcal{T}}^{(0)}(\sigma')] = 0$ and (2.8) one can write,

$$[\mathcal{T}(\sigma), \tilde{\mathcal{T}}(\sigma')] = [\mathcal{T}^{(0)}(\sigma), \Delta\mathcal{T}(\sigma')] + [\Delta\mathcal{T}(\sigma), \tilde{\mathcal{T}}^{(0)}(\sigma')]. \tag{B.13}$$

Computation of the commutators on the right hand side gives the following result:

$$\begin{aligned}
 [\mathcal{T}(\sigma), \tilde{\mathcal{T}}(\sigma')] &= \pi i \left(K \Pi^+(\sigma) \tilde{\Pi}^+(\sigma') + \frac{1}{8\pi T} \vec{\partial}^2 K \Pi^+ \tilde{\Pi}^+(\sigma') \right. \\
 &\quad \left. - K \tilde{\Pi}^+(\sigma') \Pi^+(\sigma) - \frac{1}{8\pi T} \vec{\partial}^2 K \Pi^+ \tilde{\Pi}^+(\sigma) \right) \delta'(\sigma - \sigma') \\
 &+ \frac{i}{2} \sqrt{\frac{\pi}{T}} \left(\partial_+ K (\Pi^+)^2 \tilde{\Pi}^+(\sigma) + : \vec{\Pi} \cdot \vec{\partial} K \Pi^+ \tilde{\Pi}^+(\sigma) : \right. \\
 &\quad \left. - \partial_+ K \Pi^+ (\tilde{\Pi}^+)^2(\sigma) - : \vec{\Pi} \cdot \vec{\partial} K \Pi^+ \tilde{\Pi}^+(\sigma) : \right) \delta(\sigma - \sigma'). \tag{B.14}
 \end{aligned}$$

Using the identity (B.10) and $\Pi^\mu - \tilde{\Pi}^\mu = 2\sqrt{\pi T} \partial_\sigma X^\mu$ the above expression can be simplified to:

$$[\mathcal{T}(\sigma), \tilde{\mathcal{T}}(\sigma')] = \frac{i}{8T} \partial_\sigma \left(\vec{\partial}^2 K \Pi^+ \tilde{\Pi}^+(\sigma) \right) \delta(\sigma - \sigma'). \tag{B.15}$$

This establishes eqs. (2.13), (2.10).

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